

Transport Properties

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Momentum

- The average momentum carried per particle

$$\phi = mu(y)$$

- Its transport results in a shear stress, so

$$J_{mom} = -\tau = -\frac{\alpha_\mu}{2} mn\bar{c}l \frac{du}{dy} = -\mu \frac{du}{dy}$$

- Thus

$$\mu = \frac{1}{2} \alpha_\mu mn\bar{c}l = \frac{1}{2} \alpha_\mu \rho \bar{c}l$$



Viscosity

- One can show from more advanced theory

$$\alpha_{\mu} \simeq 1$$

- Using the Ideal Gas Law

$$\mu \propto \frac{T^{1/2}}{d^2}$$

- A better approximation for viscosity is Sutherland's formula

$$\mu = 0.998 \frac{\sqrt{\pi m k T}}{\pi \sigma^2} \frac{1}{1 + S / T}$$

S is a fitting constant



Viscosity

- Units: Dynamic vs Kinematic

$$\mu \left[\frac{m}{Lt} \right] \quad \frac{\text{kg}}{\text{m sec}}, \text{ poise} = \frac{\text{gm}}{\text{cm sec}}, \text{ cp} = 10^{-2} \text{p}$$

$$\nu = \frac{\mu}{\rho} \left[\frac{L^2}{t} \right] \quad \frac{\text{m}^2}{\text{sec}}, \frac{\text{cm}^2}{\text{sec}}, \text{ etc.}$$

Gas	Absolute Viscosity (10^{-5} Pa s)								
	Temperature ($^{\circ}\text{C}$)								
	0	20	50	100	200	300	400	500	600
Air	1.73	1.82	1.96	2.20	2.61	2.98	3.32	3.64	3.94
Ammonia	0.92	0.99	1.10	1.30	1.68	2.06	2.44	2.82	3.19
Argon	2.1	2.23	2.42	2.73	3.28	3.77	4.22	4.64	5.04
Benzene	0.7	0.75	0.81	0.94	1.20				
Carbon dioxide	1.37	1.47	1.61	1.85	2.30	2.71	3.08	3.42	3.74
Carbon monoxide	1.66	1.74	1.88	2.10	2.52	2.90	3.25	3.56	3.86
Chlorine	1.23	1.32	1.45	1.69	2.10	2.50			
Chloroform	0.94	1.01	1.11	1.28	1.62	1.95			
Ethylene	0.97	1.03	1.12	1.28	1.54	1.79			
Helium	1.87	1.96	2.10	2.32	2.73	3.12	3.48	3.84	4.18
Hydrogen	0.84	0.88	0.94	1.04	1.21	1.37	1.53	1.69	1.84
Methane	1.03	1.10	1.19	1.35	1.63	1.88	2.11	2.33	2.53
Neon	2.98	3.13	3.36	3.70	4.32	4.89	5.43	5.94	6.44
Nitrogen	1.66	1.76	1.89	2.12	2.51	2.86	3.19	3.49	3.78
Nitrous oxide	1.37	1.47	1.61	1.84	2.29	2.70	3.07	3.40	3.70
Oxygen	1.95	2.04	2.18	2.44	2.93	3.37	3.76	4.13	4.47
Steam	0.92	0.97	1.06	1.24	1.62	2.03	2.45	2.86	3.26
Sulphur dioxide	1.16	1.26	1.40	1.64	2.09	2.51	2.90	3.26	3.61
Xenon	2.12	2.28	2.51	2.88	3.57	4.20	4.79	5.34	5.86

- $1 \text{ Pa s} = 1 \text{ N s/m}^2 = 10 \text{ poise} = 1,000 \text{ milliPa s}$

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Energy

- The average internal energy carried per molecule for a monatomic gas is

$$\phi = u = mc_v T$$

- The heat flux is

$$J_\phi = q = -\frac{1}{2}\alpha_\lambda n \bar{c} l m c_v \frac{dT}{dy} = -\lambda \frac{dT}{dy}$$

- Thermal conductivity

$$\lambda = \frac{1}{2}\alpha_\lambda \rho \bar{c} l c_v = \frac{\alpha_\lambda}{\alpha_\mu} \mu c_v$$



Thermal Conductivity

- Recall the Prandtl number

$$\text{Pr} = \frac{c_p \mu}{\lambda} = \frac{c_p \alpha_\mu}{\alpha_\lambda c_v}$$

- Recall

$$\gamma = \frac{c_p}{c_v} \quad \text{Pr} = \frac{\alpha_\mu}{\alpha_\lambda} \gamma$$

- For a monatomic gas

$$\frac{\alpha_\mu}{\alpha_\lambda} = \frac{\gamma}{\text{Pr}} \simeq \frac{5/3}{2/3} = \frac{5}{2} \quad \text{so} \quad \alpha_\lambda \simeq \frac{5}{2}$$



Eucken Correction for Non-Monatomic Gas

- For non-monatomic ideal gas we can write

$$u = u_{tr} + u_{int}$$

$$c_v = c_{vtr} + c_{vint}$$

- Assume

$$\lambda = \lambda_{tr} + \lambda_{int} = \frac{5}{2} \mu c_{vtr} + \mu c_{vint}$$

$$\lambda = \mu \left(\frac{5}{2} c_{vtr} + c_{vint} \right)$$

		Thermal conductivity in mW m ⁻¹ K ⁻¹						Ref.
		100 K	200 K	300 K	400 K	500 K	600 K	
	Air	9.5	18.5	26.4	33.5	39.9	46.0	1
Ar	Argon ($P = 0$)	6.3	12.4	17.7	22.4	26.5	30.3	2, 3*
BF ₃	Boron trifluoride			19.0	24.6			4
ClH	Hydrogen chloride		9.2	14.5	19.5	24.0	28.1	4
F ₆ S	Sulfur hexafluoride ($P = 0$)			13.0	20.6	27.5	33.8	5
H ₂	Normal hydrogen ($P = 0$)	68.2	132.8	186.6	230.9	270.9	309.1	6
H ₂ O	Water ($P = 0$)			18.6	26.1	35.6	46.2	7
D ₂ O	Deuterium oxide ($P = 0$)			18.2	26.6	36.3	47.6	8
H ₂ S	Hydrogen sulfide			14.6	20.5	26.4	32.4	4
H ₃ N	Ammonia			25.1	37.2	53.1	68.6	9
He	Helium ($P = 0$)	74.7	118.3	155.7	189.6	221.4	251.6	10
Kr	Krypton ($P = 0$)		6.5	9.5	12.3	14.8	17.1	11
NO	Nitric oxide		17.8	25.9	33.1	39.6	46.2	4
N ₂	Nitrogen	9.4	18.3	26.0	32.8	39.0	44.8	1
N ₂ O	Nitrous oxide		9.8	17.4	26.0	34.1	41.8	4
Ne	Neon ($P = 0$)	22.3	37.4	49.4	59.9	69.5	78.5	12
O ₂	Oxygen	9.1	18.2	26.5	34.0	41.0	47.7	1
O ₂ S	Sulfur dioxide			9.6	14.3	20.0	25.6	4
Xe	Xenon ($P = 0$)		3.7	5.5	7.2	8.8	10.3	3*, 11
CCl ₂ F ₂	Dichlorodifluoromethane			9.9	15.0	20.1	25.2	13
CF ₄	Tetrafluoromethane ($P = 0$)			16.0	24.1	32.2	39.9	5
CO	Carbon monoxide ($P = 0$)			25.0	32.3	39.2	45.7	14
CO ₂	Carbon dioxide		9.6	16.8	25.2	33.5	41.6	15

From: “Thermal conductivity of gases” by Marcia I. Huber and Allan H. Harvey, CRC Handbook of Chemistry and Physics, 92nd Ed.



Mass

- For mass the flux is a given specie.
- For multicomponent mixtures, mass transport is quite complicated.
- Here we will consider the transport in a simple binary mixture with species 1 and 2, where

$$n_1 + n_2 = n = \text{constant}$$

- Thus

$$\frac{dn_1}{dy} = -\frac{dn_2}{dy} \quad \text{and} \quad J_{n_1} = -J_{n_2}$$

Mass

- Here $\phi = n_1 / n$ the fraction of species 1 being transported by $\frac{n\bar{c}}{4}$

$$\begin{aligned} J_{n_1} &= -\frac{1}{2} \alpha_n n \bar{c} l_1 \frac{d(n_1 / n)}{dy} \\ &\simeq -\frac{1}{2} \alpha_n \bar{c} l_1 \frac{dn_1}{dy} \end{aligned}$$

- Likewise

$$J_{n_2} \simeq \frac{1}{2} \alpha_n \bar{c} l_2 \frac{dn_2}{dy}$$

Diffusion Coefficient

- The diffusion coefficient is defined by the relation

$$J_{n_1} = -D_{12} \frac{dn_1}{dy} \quad \text{Etc.}$$

- So

$$D_{12} = \frac{1}{2} \alpha_n \bar{c}_1 l_1 = D_{21} = \frac{1}{2} \alpha_n \bar{c}_2 l_2$$

Reality

- The result that $D_{12} = D_{21}$ implies that $\sqrt{m\pi}d^2$ is the same for the two gases, which may not be true unless the two gases are nearly identical.
- In fact our result, which is called Fick's Law of Diffusion, is only valid in the limit of

$$n_1 \ll n_2$$

- In general, the equations of multicomponent diffusion must be used.



Gases in large excess of air		Diffusion coefficient, D_{12} , [cm ² /s] at atmospheric pressure and given temperatures					
		0 °C	20 °C	100 °C	200 °C	300 °C	400 °C
Name	Formula	32 °F	68 °F	212 °F	392 °F	572 °F	752 °F
Argon	Ar	0.167	0.189	0.289	0.437	0.612	0.81
Methane	CH ₄		0.21	0.321	0.485	0.678	0.899
Carbon monoxide	CO		0.208	0.315	0.475	0.662	0.875
Carbon dioxide	CO ₂		0.16	0.252	0.39	0.549	0.728
Hydrogen	H ₂	0.668	0.756	1.153	1.747	2.444	3.238
Water vapour	H ₂ O	0.219	0.242	0.399	0.638	0.873	1.135
Helium	He	0.617	0.697	1.057	1.594	2.221	2.933
Sulfur hexafluoride	SF ₆			0.15	0.233	0.329	0.438

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Some Important Non-Dimensional Numbers

- Prandtl $Pr = \frac{\nu}{\alpha}$ $Pr_{air} \simeq 0.7$
- Schmidt $Sc = \frac{\nu}{D}$ $Sc_{air} \simeq 0.55$
- Lewis $Le = \frac{D}{\alpha}$ $Le_{air} \simeq 1.3$

– Thermal Diffusivity

$$\alpha = \frac{\lambda}{\rho c_p}$$

